**Exercise 1.**

The Monte Carlo method can be used to generate an approximate value of pi. The figure below shows a unit square with a quarter of a circle inscribed. The area of the square is 1 and the area of the quarter circle is pi/4. Write a script to generate random points that are distributed uniformly in the unit square. The ratio between the number of points that fall inside the circle (red points) and the total number of points thrown (red and green points) gives an approximation to the value of pi/4. This process is a Monte Carlo simulation approximating pi. Let N be the total number of points thrown. When N=50, 100, 200, 300, 500, 1000, 5000, what are the estimated pi values, respectively? For each N, repeat the throwing process 100 times, and report the mean and variance. Record the means and the corresponding variances in a table.

蒙特卡洛方法可以用于产生接近pi的近似值。图1显示了一个带有1/4内切圆在内的边长为1的正方形。正方形的面积是1，该1/4圆的面积为pi/4。通过编程实现在这个正方形中产生均匀分布的点。落在圈内（红点）的点和总的投在正方形（红和绿点）上的点的比率给出了pi/4的近似值。这一过程称为使用蒙特卡洛方法来仿真逼近pi实际值。令N表示总的投在正方形的点。当投点个数分别是20, 50, 100, 200, 300, 500, 1000, 5000时，pi值分别是多少？对于每个N，每次实验算出pi值，重复这个过程20次，并在表中记下均值和方差。

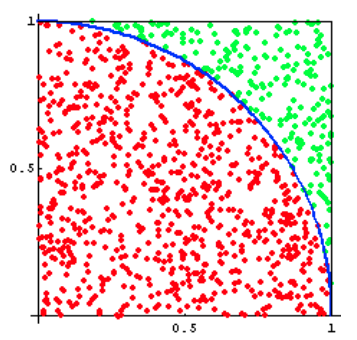


Figure 1 蒙特卡洛方法求解pi

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| COUNT\次数 | 1 | 2 | 3 | 4 | 5 |
| 20 | 2.8 | 2.4 | 2.6 | 3.2 | 3 |
| 50 | 3.2 | 3.68 | 3.2 | 2.8 | 3.2 |
| 100 | 3 | 3.04 | 3.4 | 3.32 | 3.36 |
| 200 | 3.24 | 3.24 | 3.08 | 3.08 | 2.92 |
| 300 | 3.08 | 3.10667 | 3.34667 | 3.21333 | 3.10667 |
| 500 | 3.08 | 3.264 | 3.304 | 3.048 | 3.184 |
| 1000 | 3.108 | 3.056 | 3.128 | 3.212 | 3.104 |
| 5000 | 3.172 | 3.1696 | 3.104 | 3.1384 | 3.112 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 6 | 7 | 8 | 9 | 10 | 11 |
| 3 | 3.2 | 2.4 | 3 | 2.8 | 3.2 |
| 3.36 | 2.88 | 2.88 | 3.04 | 2.96 | 3.44 |
| 3.12 | 2.8 | 3 | 3.4 | 3.48 | 3.16 |
| 3.12 | 3.02 | 3.2 | 3.16 | 3.14 | 3.02 |
| 3.17333 | 3.06667 | 2.90667 | 3.21333 | 3.2 | 3.4 |
| 3.168 | 2.968 | 3.168 | 3.152 | 3.104 | 3 |
| 3.16 | 3.18 | 3.116 | 3.172 | 2.076 | 3.152 |
| 3.152 | 3.16 | 3.1328 | 3.1328 | 3.128 | 3.1624 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 12 | 13 | 14 | 15 | 16 | 17 |
| 3.4 | 3 | 3.6 | 3 | 2.2 | 3.6 |
| 3.12 | 2.96 | 2.96 | 2.88 | 3.6 | 3.12 |
| 3.08 | 3.12 | 3.04 | 3.28 | 3 | 3.28 |
| 3.24 | 3.26 | 3.28 | 3.14 | 3 | 3.24 |
| 3.09333 | 3.04 | 3.14667 | 2.93333 | 3.13333 | 3.09333 |
| 3.096 | 3.04 | 3.24 | 3.048 | 3.064 | 2.968 |
| 3.108 | 3.124 | 3.2 | 3.144 | 3.208 | 3.152 |
| 3.1632 | 3.1368 | 3.16 | 3.1256 | 3.124 | 3.1248 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 18 | 19 | 20 | 方差 | 均值 |
| 3.8 | 3 | 2.8 | 0.164 | 3 |
| 3.36 | 2.96 | 3.12 | 0.057984 | 3.136 |
| 2.96 | 3.16 | 3.08 | 0.030124 | 3.154 |
| 3.1 | 3.18 | 3.32 | 0.010819 | 3.149 |
| 3.10667 | 3.10667 | 3.2 | 0.012675524 | 3.1333335 |
| 3.096 | 3.104 | 3.256 | 0.00904384 | 3.1176 |
| 3.12 | 3.176 | 3.068 | 0.05567116 | 3.053538462 |
| 3.136 | 3.1696 | 3.1848 | 0.000471198 | 3.14444 |